On a deterministic disguise of orthodox quantum mechanics III

Version 3, October 2016

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According to quantum theory, pure physical states correspond to equivalence classes of state vectors, where any two members of one class differ by a complex factor. The point is that such a factor does not change the probability for the occurrence of any measurement result as computed within the formalism of quantum mechanics. In the formalism to be presented here, the state vector does not only determine the probabilities for the occurrence of measurement results, rather it determines the result itself. The information which guides the selection process is obtained from the state vector in a way that is not invariant against multiplication by a complex factor. It can be seen as a kind of global phase of the state vectors. This global phase then is a perfectly hidden variable. For this presentation of quantum mechanics we have a remarkable parallelism to classical statistical mechanics: If we would know exactly the initial state of a system, we could compute the outcomes for all experiments to be done later on this system. In the quantum case the initial state is to be represented by its state vector with specified phase, e.g. as a concrete complex-valued wave function.

1 Introduction

As is well known, quantum mechanics can be interpreted in a number of ways. The view on quantum mechanics which underlies the present work is similar to the 'Copenhagen spirit' as outlined in chapter VII.1 of [1]. See also [2], Ch. IV, §§ 15-19. We thus deal with systems that are amenable to reproducible *experiments* consisting of manipulations and observations that can be described in an unambiguous language [1]. As N. Bohr put it (cited from [1], p. 295): 'We must be able to tell our friends what we have done and what we have learned'.

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[1] Concepts such as the 'wave function of the universe' are outside this scope.

It has tradition to characterize this unambiguous language as the language of classical physics. In my eyes it is clearer to characterize it as the language of engineering. For instance, modern photo-detectors such as CCDs are based on non-classical concepts of semiconductor physics. However, as commercially available products, these are the outcome of documented manufacturing processes, based on documented designs, guided by a documented device theory. This device theory is based on authoritative textbooks such as [3] which don't speak the 'quantum language' with states, Hamilton operators, and observables.

The existence of an unambiguous language is paralleled by the existence of unambiguous actors: electrons, photons, atoms, molecules and alike. Despite all quantum uncertainty associated with motion and internal dynamics of these quantum actors, their constant physical properties like rest-mass, charge, and internal angular momentum, together with their persistence (each of the atomic nuclei around us has an individual history going back some billion years to its formation in the interior of some no longer existing star) allow us to perform experiments which deliver reproducible statistical data derived from myriads of individual factual measuring events. A typical case of such measuring events are detector 'clicks' which say where a particle was found. Assume that the detectors belong to a conventional scattering experiment, where a beam of particles penetrates a target foil and the scattered particles get registered. Then it is clear that the detected impacts allow an unambiguous digital documentation. It is also clear that quantum theory gives no recipes for predicting the individual impacts. It is only the probability for finding an impact at an arbitrarily chosen position for which quantum mechanics allows to derive formulas.

The idea that the physical content of a motion theory of atomic and subatomic particles could be intrinsically statistical is not natural for most physicists. When the interpretation of Schrödinger's $|\psi|^2$ as a probability emerged from papers by M. Born and others (see. [4]) it created disbelief and opposition such as Einstein's (abridged) remark 'God does not play dice'. Against Leibniz's 'principle of sufficient reason' the impacts should happen once here then there and some miraculous principle should tame their arbitrariness as to observe the probabilistic rules which are the content of theory. Never before was a theory proposed of that kind. Notice that in classical statistical mechanics of gases each molecular collision is determined by the locations and velocities of nearby molecules or walls. The statistical character comes only from the fact that the number of dynamical variables is too large for a treatment on the fundamental level. With the employment of computers, working on the fundamental level gained large new territories [5], though. Determinism and causality as claimed above for collisions allow time-evolution to run autonomously. No authority is required to enforce obedience to the probabilistic rules while still respecting conservation laws. Rather, the process evolves in the manner of an algorithm. This does not imply that future states of the process can be predicted; strong dependence on initial conditions may make predictions fail.

These considerations suggest the question whether a logically equivalent situation could hold for quantum mechanics as well. One answer to this question is known: D. Bohm's deterministic version of quantum mechanics [6] augments the Schrödinger wave function with particles having definite positions and having momenta that are defined

in terms of the wave function. The deterministic time evolution of the wave function forces the particles to follow according to a deterministic rule. The wave function is said to act as a guiding wave for the particle. A particle detector 'click' is assumed to indicate just the position of such a guided particle. Whether a detector will 'click' or not thus is determined by the 'Bohmian state' since it specifies the positions of the particles. Although it is thus a theory with 'hidden variables' (the particle positions) in a general meaning of the words, it is well understood today (cf. [7]) that it is not covered by von Neumann's no-go theorem on hidden variable theories. The presently proposed method is literally a hidden variable theory as well — the global phase of the wave function is the single hidden variable. Since it makes exactly the prediction of quantum mechanics, any no-go theorem which would exclude the present proposal had also to exclude quantum mechanics.

2 Measurements

With respect to mathematical representations of measurements and states we follow conventional lines: A complex Hilbert space $\mathcal{H}$, the *normalized* elements of which will always be denoted by ψ and referred to as *state vectors* or *wave functions*, allows representing (pure) *states* as equivalence classes of state vectors with respect to the relation

$$\psi \sim \psi' \quad \Leftrightarrow \quad \exists\, \omega \in \mathbb{C}_1 \;\; \psi' = \omega\psi \;, \qquad \text{where} \quad \mathbb{C}_1 := \{z \in \mathbb{C} \;:\; |z| = 1\} \;. \tag{1}$$

Wave functions are considered time-dependent according to the Schrödinger picture, where time refers to an inertial system of reference which is assumed to be selected once and for all. This implies that whenever a specific wave function ψ is under consideration, there is some $t_\psi \in \mathbb{R}$, the point in time for which ψ is valid.

Since all Hilbert spaces of the same dimension are isomorphic, no specific features of an experiment are expressed by selecting $\mathcal{H}$. A connection to the physical world is made by giving certain linear operators in $\mathcal{H}$ a meaning as descriptors of entities ranging from observable quantities to measurement devices, dynamical variables, and symmetries.

Measurement devices are represented by self-adjoint operators in $\mathcal{H}$. Self-adjoint operators viewed as representing measurement devices will be called *observables*.

Since measurements always have a limited resolution and a finite operational range it is justified to consider as observables only those self-adjoint operators which have a spectrum [2] consisting of finitely many points [3]. The general form of an observable A thus is

$$A = \sum_{a \in \sigma_A} a\, P_a \;, \tag{2}$$

where σ_A is a finite subset of $\mathbb{R}$ and $(P_a)_{a\in\sigma_A}$ is a family of pairwise orthogonal projectors which add up to the identity operator in $\mathcal{H}$. The expression on the right-hand side of equation (2) is said to be the *spectral representation* of A. Observables in the sense of (2)

[2] this, of course, is the set of possible outcomes of measurements

[3] Self-adjoint operators that primarily act as generators of unitary one-parameter groups, such as the Hamiltonian, are excluded from this restriction.

fall short of modeling most realistic measurements as they are restricted to giving a single number as a result. This precludes them e.g. from recording the direction of of a particle track in space, or, even more so, the tracks of a group of particles. A sufficient degree of generality is reached if we consider lists $(A_1, \ldots, A_n), n \in \mathbb{N}$, of mutually commuting observables [4]. When interacting with a state vector ψ, such a *composite observable* outputs a list $(a_1, \ldots, a_n)$ as a result, where $a_i \in \sigma_{A_i}$. This list can be considered as the result of a single measurement obtained with a complex experimental setup, which may, for instance, comprise some spatially separated particle detectors and analyzers. The a_i may show dependencies, such as $a_1 = -a_2$ due to relations between the operators, such as $A_1 = -A_2$, or, less trivially, due to algebraic properties of state vector ψ as in the case of the singlet state in Section 3. A composite observable which cannot be extended in a non-trivial manner [5] is said to be a *complete set of commuting observables*. This is a standard concept of quantum mechanics, whereas the name composite observable is not in common use.

We now turn to constructing a function μ which takes as arguments an arbitrary composite observable $(A_1, \ldots, A_n)$ and an arbitrary $\psi \in \mathcal{H}$ and gives as a result a list $(a_1, \ldots, a_n) \in \sigma_{A_1} \times \ldots \times \sigma_{A_n}$. In order to simplify indexing we actually consider only the case $n = 2$ as a pattern for $n \in \{1, 2, 3, \ldots\}$. Instead of (A_1, A_2) we write (A, B), where

$$B = \sum_{b \in \sigma_B} b\, Q_b \; . \tag{3}$$

Our assumption $AB = BA$ can be shown to be equivalent to $P_a Q_b = Q_b P_a$ for all $a \in \sigma_A$ and all $b \in \sigma_B$. In such a setting the two spatially separated spin measurements which are considered in D. Bohm's discussion of the EPR-experiment (see e.g. [6], Chapter 7, [8], Chapter 3.9 and Chapter 6.3), and that are the topic of Section 3, can be treated as a single measurement with a single result (consisting of a pair of values). That observables corresponding to spatially separated measuring devices commute is a foundational principle of local quantum theory [1] and of quantum field theory [9].

Our composite observable (A, B) determines the set $\sigma_A \times \sigma_B$ which contains all potential [6] measurement results. Which result is to be expected in an actual measurement depends on the state vector ψ of the system at the instant of measurement. Quantum mechanics says that the probability $p(a, b)$ for getting the result (a, b) is given by

$$p(a, b) := \langle\, \psi \,|\, P_a Q_b \psi \,\rangle \; . \tag{4}$$

This probability can be written as a *transition probability*

$$p(a, b) = |\langle\, \psi \,|\, \psi(a, b) \,\rangle|^2 \tag{5}$$

[4] To consider the W^*-algebra generated by $A_1, \ldots, A_n$ may be considered more canonical. This, however, introduces a degree of sophistication that we want to avoid.

[5] Trivial extensions add self-adjoint operators which are functions of the operators which belong the composite observable already.

[6] If $B = f(A)$, not all $(a, f(a')), a, a' \in \sigma_A$ are possible measurement results, only $(a, f(a)), a \in \sigma_A$ are.

by means of the 'collapsed wave function'

$$\psi(a,b) := \frac{P_a Q_b \psi}{\| P_a Q_b \psi \|} \ . \tag{6}$$

If the system is still existing after a measurement that gave the result (a,b) and an immediately subsequent measurement is feasible, then the state vector for this measurement has no longer to be taken as ψ, but as $\psi(a,b)$. This statement is normally referred to as the *projection postulate*.

It is instructive to see that assuming the corresponding relation for single observables instead of pairs, allows us to deduce the result for pairs and, actually, arbitrarily long lists by successive application:

$$\psi[b] := \frac{Q_b \psi}{\| Q_b \psi \|} \ , \quad \psi[a,b] := \frac{P_a \psi[b]}{\| P_a \psi[b] \|} = \frac{P_a Q_b \psi}{\| Q_b \psi \| \, \frac{\| P_a Q_b \psi \|}{\| Q_b \psi \|}} = \psi(a,b) \ . \tag{7}$$

This approach is, however, less natural than the one we started with. This is especially so if our measurement device contains spatially separated subsystems α and β, which are responsible for creating the partial results a and b respectively. Then, we are forced to give the appearance of the partial results a causal order. Here, in accordance with (7), we let b appear first. Then, we need to appeal to the projection postulate and employ the state $\psi[b]$ for computing the the probability for a which is consistent with (4) for the pair (a,b) of results. The state relevant for the computation thus seems to switch from ψ at the place of β to $\psi[b]$ at the place of α. This, however, is based on a misconception concerning the localization of the effectuality of states. That the relation between a and b has nothing to do with causality can be seen from the fact that one could reverse the roles of a and b which then would suggest a causal connection in the opposite direction.

In conclusion, the physical situation as represented by the measurement device (A,B) and the state vector ψ assigns to each element $(a,b) \in \sigma_A \times \sigma_B$ a probability $p(a,b)$ as given by (4). The act of measurement selects one of the elements of $\sigma_A \times \sigma_B$ in a way which is compatible with these probabilities. Of course, not always an element with a major assigned probability will be selected; only a probability zero is prohibitive for becoming selected. The predictive power of the theory for a single measurement is, therefore, rather poor. If, however, the physical situation allows the measurement to be repeated with the relevant conditions unchanged, statistics over the selected values can be done and the consistency of the obtained measurement data and the the probability measure p can be quantified.

Taking quantum mechanics with M. Born as an inherently statistical theory the selection of the actual measurement value out of the manifold of possible values is a 'true' random event. Replacing this random event by an algorithmically generated, thus deterministic, pseudo-random event brings about the *deterministic disguise* announced in the title.

Whereas in biological systems there are many well-understood mechanisms that have a (pseudo-)random output (e.g. the distribution of the parent genes onto the descendants), it is hard to hypothesize how pseudo-random generators in non-living quantum

systems could be organized. Here I assume as a working hypothesis that quantum mechanics (actually all Nature) builds on some unknown algorithmic infrastructure [7]. In the absence of any pertinent knowledge it is a natural assumption that the computational strength of this structure is just so that it allows to compute all functions which are *computable* in the technical sense and that, therefore, a valid pseudo-random number generator (RNG) can be realized within this infrastructure (see e.g. [10] Chapter 7, The Intrinsic Generation of Randomness). We thus take for granted that a RNG is a legitimate building block of a physical theory, much like integrals and differential equations.

More specifically, we assume a RNG which outputs uniformly distributed real numbers in the interval $[0, 1)$. The meaning of this is a pragmatic one: the generator passes a reasonable set of statistical tests of the hypothesis that the output of the generator comes from independent repetition of a stochastic experiment which outputs a value $\xi \in [0, 1)$ such that the event $\xi \in [a, b) \subset [0, 1)$ has probability $b - a$. Or, as it is expressed in [12], p.1: 'Here we suppose that the goal is that successive output values of the RNG, say $u_0, u_1, u_2, \ldots$, imitate independent random variables from the uniform distribution over the interval $[0, 1]$.'

A simple example is the RNG [11]

$$u_i := 1000000 \sin(i) - \lfloor 1000000 \sin(i) \rfloor \tag{8}$$

which can be shown to pass all 15 tests of the test battery SmallCrush of TestU01 version 1.2.3 (see [12]) and fails in 20 of 144 tests of the more demanding battery Crush, and in 44 of 160 in the even more demanding BigCrush. The latter consumed 16.5 hours of CPU time on a dual core 2.4 GHz Laptop Computer.

Forming weighted means out of several RNGs gives again a RNG with typically better statistical properties. The situation seems to be that for each test battery there can be constructed a RNG which passes it completely and for each RNG there can be constructed a test battery which lets it fail. The real challenge is to identify in this infinite competition a David versus Goliath situation, where a relatively simple RNG passes a huge, complex, and powerful test battery.

The value-range $[0, 1)$ for RNGs is standard in programming languages and comes in naturally in algorithms such as (8). In some of the following considerations the toroidal value range $\mathbb{C}_1$ would be more natural in the first place. The bijection

$$\tau \,:\, [0, 1) \,\to\, \mathbb{C}_1 \,, \quad \xi \quad \mapsto \quad \exp(2\pi \,\mathrm{i}\, \xi) \tag{9}$$

will provide the connection where needed.

In programming languages a random generator is normally a named function (such as `rand()` in the language C) which needs no argument to yield a value. This is, in a sense, a fallacy since the function has a counter as an internal state which gets incremented by each function call and which works as a hidden argument. When using pseudo-random

[7] A plausible speculation sees the update rules of a universal discrete causal network in this role. Since nothing specific from this body of ideas is needed in the present work, a single reference: [10] Chapter 9 and Notes for Chapter 9 may suffice.

generators as legal citizens of theoretical physics one should give them a more natural interface, which does not rely on the a concept of individuality as witnessed by a state variable. Instead we take for granted 'pseudo-random clocks'

$$\chi \,:\, \mathbb{R} \,\to\, [0,1)\,, \qquad \chi_\tau \,:\, \mathbb{R} \,\to\, \mathbb{C}_1\,, \quad \chi_\tau := \tau \circ \chi\,, \tag{10}$$

which associates a pseudo-random number and a pseudo-random phase with any point in time. Just as numerical values of time are not provided by Nature but by a man-made time-keeping system (which has, however, to satisfy conditions set by Nature), we consider also the numerical output of the random clocks as a part the man-made time-keeping system. Actually, on the Internet there are various services which provide both pseudo-random numbers and 'true' random numbers (derived from natural processes such as radioactivity) on demand.

Pseudo-random numbers can be used to determine rather general pseudo-random events by constructing images of the probability space to which the pseudo-random numbers belong. In our case we construct the mapping

$$\rho_\psi \,:\, [0,1) \,\to\, \sigma_A \times \sigma_B \tag{11}$$

such that the image of the Lebesgue measure λ on $[0,1)$ under ρ_ψ is just p from (4):

$$\rho_\psi(\lambda) = p\,. \tag{12}$$

The construction of ρ_ψ suggests itself. Define

$$X := \{(a,b) \in \sigma_A \times \sigma_B \,|\, p(a,b) \neq 0\}$$

and write it in lexicographic order as

$$X =: \{x_1, \ldots, x_n\}\,.$$

Define $\{y_0, y_1, \ldots, y_n\}$ iteratively by $y_0 := 0$, $y_i := y_{i-1} + p(x_i)$; obviously $y_n = 1$. Defining the intervals $I_i := [y_{i-1}, y_i)$ we have the partition

$$[0,1) = I_1 \cup \cdots \cup I_n$$

and define ρ_ψ as constant on each of the intervals I_i and taking the value x_i on I_i.

For any $[0,1)$-valued λ-distributed random variable x the random variable $\rho_\psi \circ x$ is p-distributed. Notice that the connection of measures λ and p does deal only with measures and a mapping; it needs neither random variables nor RNGs.

One way to complete our construction of function μ would be to require each measurement in state ψ at time t_ψ to request the random number $\xi := \chi(t_\psi)$ from the random clock and to stipulate that the outcome (a,b) of the experiment be given as $\rho_\psi(\xi)$. The first sentence in the last preceding paragraph then implies that this rule let the measurements, when analyzed statistically, appear indistinguishable from measurements that result from the intrinsically statistical version of quantum mechanics.

In this method one may criticize the appearance of t_ψ on grounds of the possibility that for many measurements the trigger for measurement is not in the device but in the system to be measured (e.g. in the collision of two particles). Since the formation of the actual measurement values is determined by ξ we need ξ for an unknown time which would destroy the determinism of our scheme. It is desirable to refer to t_ψ only at the end of a measurement, where the measurement result and the point in time exist in documented form and belong to the world that can be represented in the 'engineering language'. The method to be proposed in the following achieves this.

It derives ξ from the state vector $\psi \in \mathcal{H}$. The simplest choice is $\xi = \Theta(\psi)$, where Θ is a function $\mathcal{H} \to \mathbb{C}_1$ such that $\Theta(\omega\psi) = \omega\Theta(\psi)$ for all state vectors ψ and all $\omega \in \mathbb{C}_1$. There are many such functions and without having input from a special situation we have to rely on arbitrary selections. Here we choose an orthonormal base $(\phi_i)_{i=1}^d, d \in \mathbb{N} \cup \{\infty\}$, in $\mathcal{H}$ and set for each state vector ψ

$$k_\psi = \min\{k \in \mathbb{N} \,:\, \sum_{i=1}^{k} |\langle\, \phi_i \,|\, \psi \,\rangle|^2 > 1/2\} \,. \tag{13}$$

Among the finitely many complex numbers $\langle\, \phi_i \,|\, \psi \,\rangle$, $1 \le i \le k_\psi$, we select the one of greatest modulus and name it z. Then

$$\Theta \,:\, \mathcal{H} \,\to\, \mathbb{C}_1 \,, \quad \psi \quad \mapsto \quad \frac{z}{|z|} \,. \tag{14}$$

The final formula for the measurement result (a, b) and the desired function μ (see p. 4) is

$$(a, b) = \rho_\psi(\tau^{-1}(\Theta(\psi))) =: \mu((A, B), \psi) \,. \tag{15}$$

In order for this to work we have to manage the circumstances that determine the phase of a state vector. A state comes into existence either by preparation or assumption (typically based on a priori knowledge). Let us consider an experiment dealing with a single isolated hydrogen atom. The state of it is 'self-preparing'; after some time it can be assumed to be the ground state. Physicists know the formula for the hydrogen atom ground state wave function. No question that in Nature there occurs something that corresponds closely to the behavior of the mathematical object defined by this formula. For mathematical deductions concerning future behaviors of the object it suggests itself to take the wave function as given by the books as initial state vector. Actually, any multiple by a $\omega \in \mathbb{C}_1$ would represent the same state. Since in the formalism to be presented here, the overall phase of the wave function *does* matter, it would be unnatural to rely here on a choice that text book authors made, based on their understanding of simplicity of formulas. Even in the unmodified quantum mechanics formalism we have to acknowledge that the time-dependent Schrödinger equation deals with state vectors and not with states. If we understand time-evolution as an algorithm, as indicated earlier in this section, it is clear that this algorithm works on a state vector and not on a state. The logic is similar to an algorithmic implementation of the arithmetic of integers: one has to select a base (e.g. 2 or 10) although the results depend on this choice in an obvious

way which allows to switch from one representation to another. Let this be motivation enough for the following rule: Whenever a state vector is to be introduced into the description of an experiment, there is a 'time of birth' t^* for which the preparation of the state was finished. Then the 'newly born' state vector has to be given a phase such that

$$\Theta(\psi) = \chi_\tau(t^*) \ , \tag{16}$$

with χ_τ from (10).

In a computational model of a quantum system formulated in C++ (and in many other object oriented programming languages) an object comes into existence through the execution of a constructor function. In such a framework one simply has to make this random phase operation (16) the last statement in all state constructors. Also the state vector (6) has to be considered 'newly born' and has to be 're-phased' as to satisfy

$$\Theta(\psi(a,b)) = \chi_\tau(t) \ , \tag{17}$$

where t is the time of completed measurement.

Let us summarize the properties of the proposed modification of orthodox quantum mechanics:

1. The result of every measurement is strictly determined.
2. All measurement results occur with the probability predicted by quantum theory.
3. Since each measurement introduces new randomization of the phase, there is no way to experimentally find out the mechanism behind the built-in determinism.
4. Determinism is enforced by the same means that a programmer would employ in creating a simulation of chained quantum measurements.

3 A computational low-dimensional model

As a proof of concept I implemented the ideas presented so far as an interactive free computer program on the web [13]. The source code of the program is freely accessible. It is written in *Wolfram Language* which is a very expressive universal programming language which, due to its freely accessible documentation, can well serve as a replacement for expressing algorithmic content in pseudo-code.

The system under consideration is a simplified EPR experiment with two electron spins. By running this program one observes how the simulated measurement results approach the quantum mechanical exact result for the spin-spin correlation coefficient. The present form of an article does not allow to present such direct evidence. Nevertheless, a description of the system and the simulated results from a single arbitrarily selected run may be a useful illustration of the method described in Section 2.

The model is defined by Hilbert space and operators as follows:

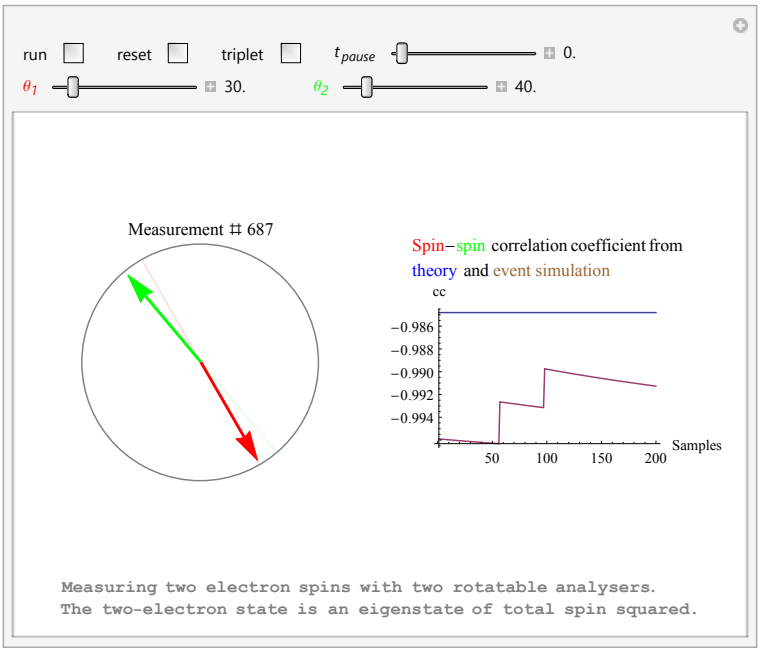

Figure 1: Simulated measurements of two electron spins along selectable directions.

$$
\begin{aligned}
\mathcal{H} &:= \mathbb{C}^2 \otimes \mathbb{C}^2 \,, \\
s_x := \frac{\sigma_x}{2} \,, &\quad s_y := \frac{\sigma_y}{2} \,, \quad s_z := \frac{\sigma_z}{2} \\
s_x^1 := s_x \otimes \mathbf{1} \,, &\quad s_y^1 := s_y \otimes \mathbf{1} \,, \quad s_z^1 := s_z \otimes \mathbf{1} \,, \\
s_x^2 := \mathbf{1} \otimes s_x \,, &\quad s_y^2 := \mathbf{1} \otimes s_y \,, \quad s_z^2 := \mathbf{1} \otimes s_z \,, \\
S_x := s_x^1 + s_x^2 \,, &\quad S_y := s_y^1 + s_y^2 \,, \quad S_z := s_z^1 + s_z^2 \,, \\
S^2 &:= S_x^2 + S_y^2 + S_z^2 \,, \\
U(\theta_1, \theta_2) &:= \exp(\mathrm{i}\,\theta_1 s_z) \otimes \exp(\mathrm{i}\,\theta_2 s_z) \,,
\end{aligned}
\tag{18}
$$

where, of course, $\sigma_x, \sigma_y, \sigma_z$ denote the Pauli matrices.

All these operators are actually square matrices and their spectral representation can be obtained by numerical methods such as the function `Eigensystem` of the computer algebra system Mathematica. Of course, for the operator S^2 we get the eigenvalues 0 and 2 together with two projectors onto an 1-dimensional and a 3-dimensional state space, corresponding to what is known as singlet and triplet states respectively. We select a singlet state ψ which is defined uniquely up to a phase factor. With the system assumed to be in this state we simulate the measurement of the composite observable (A, B) with

$$
A := U(\theta_1, \theta_2)\; s_x^1\; U(-\theta_1, -\theta_2) \,, \quad B := U(\theta_1, \theta_2)\; s_x^2\; U(-\theta_1, -\theta_2) \,. \tag{19}
$$

When running, the program executes the command [8]

$$(a, b) = \mu(A, B,\, \mathrm{e}^{2\pi\,\mathrm{i}\,\mathrm{RandomReal}[\,]}\,\psi) \tag{20}$$

repeatedly, creating a list

$$(a_1, b_1), (a_2, b_2), (a_3, b_3), \ldots, (a_n, b_n)\ , \quad a_i, b_i \in \{1/2, -1/2\}\ , \tag{21}$$

where the repetition stops if some parameter, such as the state ψ or the angles θ_1, θ_2 get changed via the graphical user interface. The most recent result $(a_n, b_n) = (-1/2, 1/2), n = 687$ is shown on the left-hand side of Figure 1 as a red and a green arrow which connect the origin $(0, 0)$ with points

$$(a_n \cos(\theta_1 + \frac{\pi}{2}),\ a_n \sin(\theta_1 + \frac{\pi}{2}))\ (\text{red})\ , \quad (b_n \cos(\theta_2 + \frac{\pi}{2}),\ b_n \sin(\theta_2 + \frac{\pi}{2}))\ (\text{green})\ . \tag{22}$$

The complicating addition of $\pi/2$ is needed since the vertical (and not the horizontal) direction was chosen to correspond to $\theta \in \{0, \pi\}$. In the example shown, the two angle settings differ only by a small angle of 10° so that there is a probability close to 1 that a_n and b_n differ in sign and, thus, the arrows point in nearly opposite directions.

The graphics on the right-hand side deals with the spin-spin correlation coefficient [9]. The jagged brownish curve shows the most recently obtained 200 values of the correlation coefficient of the obtained data:

$$c_{n-199}, c_{n-198}, \ldots, c_{n-1}, c_n\ , \tag{23}$$

where the c_i are given by

$$c_i := \frac{\sum_{j=1}^{i} a_j b_j}{\sqrt{\sum_{j=1}^{i} a_j^2}\,\sqrt{\sum_{j=1}^{i} b_j^2}}\ . \tag{24}$$

Each of the jags indicates an event with $a_i = b_i$ (recall that $a_i = -b_i$ is much more probable). In all the countless runs I listened, the brownish curve eventually reached the theoretical value (25), indicated by the blue horizontal line and continued wiggling around this curve. Sometimes one observes that rather pronounced deviations from the blue line build up. But, as a rule, they have a smaller amplitude as in earlier deviation episodes.

The exact quantum mechanical value for the spin-spin correlation coefficient is

$$\frac{\langle\, \psi \,|\, A\, B\, \psi \,\rangle}{\sqrt{\langle\, \psi \,|\, A^2\, \psi \,\rangle}\,\sqrt{\langle\, \psi \,|\, B^2\, \psi \,\rangle}}\ . \tag{25}$$

For ψ being the singlet state this quantity can easily be seen to equal $-\cos(\theta_1 - \theta_2)$ and thus to yield perfect anti-correlation for $\theta_1 = \theta_2$.

[8] See (15) for μ. For the pseudo-random generator the name from Wolfram language [14] is used.

[9] We use a simplified definition of a correlation coefficient which assumes that the expectation value of the quantities to be correlated is zero.

4 Conclusion

It is rather evident that the method applied here can be generalized as to transform any probabilistic theory into a deterministic one. The method of converting probabilities into events is, in a sense, a reversion of the program of probability theory, which tries to capture the characteristics of streams of events by ascribing probabilities to them.

Although we have shown that it does not conflict with logic to assume a deterministic mechanism behind the emergence of measurement results, one cannot deny that the algorithmic implementation given above does not look like a strategy that Nature would follow. The algorithmic understanding of physical processes is in its infancy at best, as is the understanding of the required resources.

5 Acknowledgements

I would like to thank D.P.L. Castrigiano for his critical reading of the first version of this article, which led to lots of reformulations and additions, and even to the correction of a mathematical error. The definition of Θ contained an infinite sum which was not guaranteed to exist. A new definition of Θ avoids the problem.

References

[1] R. Haag: Local Quantum Physics, Fields, Particles, Algebras, Second Edition, Springer 1996

[2] A. Messiah: Quantum Mechanics, Vol. I, North-Holland Publishing Company 1961

[3] S.M. Sze: Physics of Semiconductor Devices, Second Edition, Wiley 1981

[4] A. Pais: Max Born and the Statistical Interpretation of Quantum Mechanics
`http://cds.cern.ch/record/141137/files/cer-000052203.pdf`

[5] Loup Verlet: Computer "Experiments" on Classical Fluids I. Thermodynamic Properties of Lennard-Jones Molecules. Phys. Rev. Vol. 159, Number 1, 5 July 1967, pp. 98-103

[6] D. Bohm, B.J. Hiley: The Undivided Universe, Routledge, 1993

[7] Jeffrey Bub: Von Neumann's 'No Hidden Variables' Proof: A Re-Appraisal.
arXiv: 1006.0499 [quant-ph]

[8] J. J. Sakurai: Modern Quantum Mechanics, Revised Edition, Addison-Wesley 1994

[9] S. Schlieder: Einige Bemerkungen zur Zustandsänderung von relativistischen quantenmechanischen Systemen durch Messungen und zur Lokalitätsforderung. Commun. math. Phys. 7, 305-331 (1968). Freely available from www.projecteuclid.org

[10] Stephen Wolfram: A New Kind of Science, Wolfram Media, Inc, 2002

[11] "Testing an Intuitive Random Generator with the Task of Emptying a Cube" from The Wolfram Demonstrations Project.
http://demonstrations.wolfram.com/TestingAnIntuitiveRandomGeneratorWithTheTaskOfEmptyingACube/
Contributed by: Ulrich Mutze (published 2010-06-15)

[12] Pierre L'Ecuyer and Richard Simard: TestU01, A Software Library in ANSI C for Empirical Testing of Random Numbers Generators, User's guide.

[13] Ulrich Mutze
"Monte Carlo Simulation of Two-Electron Spin Correlations"
http://demonstrations.wolfram.com/MonteCarloSimulationOfTwoElectronSpinCorrelations/
Wolfram Demonstration Project
Published: April 20, 2015

[14] http://reference.wolfram.com/language/ref/RandomReal.html